Background knowledge for the Topology Oberseminar at the WWU Münster

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This list is supposed to serve as a general Leitfaden for what we will assume as preknowledge for the Topology Oberseminar at the WWU Münster. We have grouped the material into several categories for which we are less and less serious about actually assuming this. We have aimed to give references to make it possible to learn the material. This list is based on a similar list that was written together with P. Teichner and M. Kreck for the MPIM Oberseminar.

1 Basic Algebraic Topology

The material in this section is considered to be the absolute core of what a research topologist has to know. We just list entire books. Of course no one will actually know every detail in all of these books.

- Hatcher, *Algebraic Topology*, [Hat02].
- Bredon, *Topology and Geometry*, [Bre93].
- Milnor–Stasheff, *Characteristic classes*, [MS74].
- Atiyah, *K-theory*, [Ati89].
- Bott–Tu, *Differential forms in algebraic topology*, [BT82].
- Husemöller, *Fibre bundles*, [Hus94].

There are of course a lot of good alternatives to these classic books and these will do as well. The books should just be considered as placeholders for the material that is covered.

2 More specialized basic topics

Here we list some more specialized topics but which we nevertheless consider to be very important. Everyone should at least have a rough idea. We also give references but these are less standard and are just meant as a recommendation.

1. Homological algebra and spectral sequences
   Cartan–Eilenberg [CE99], Weibel [Wei94], Gelfand–Manin [GM03]

2. Categories and sheaves
   MacLane [Mac71], Iversen [Ive84]
3. Riemannian geometry
   Do Carmo [dC92], Sharpe [Sha97]
4. Lie groups, Lie algebras, principal bundles and connections
   Bröcker–tom Dieck [BtD85], Dupont [Dup78]
5. Stable homotopy theory (i.e. spectra)
   Adams, Part III [Ada95]
6. Simplicial and abstract homotopy theory
   Goerss–Jardine [GJ99], Dwyer–Spalinski [DS95], May [May92]
7. Geometric group theory
   Löh [Löh17]

3 Advanced topics

Here we have some more advanced topics. These are mostly chosen to be compatible with the areas of research followed in Münster. It is the goal that everyone after being here for a longer time has at least a rough idea what the ideas and goals in these areas are. If there are guests then it is very likely that their research will fall into one of these categories and that they will have to assume some background knowledge. For all of these topics we should have local experts and part of the Oberseminar will be that the local experts give overview talks about these areas and try to sketch fundamental ideas, motivations and techniques.

1. The Atiyah Singer index theorem
2. Exotic spheres, surgery and algebraic L-theory
3. Whitehead torsion and the s-cobordism theorem
4. Thom spectra, geometric bordism and cobordism categories
5. Formal group laws and Landweber exactness
6. Algebraic K-theory
7. ∞-categories and higher algebra
8. Rational homotopy theory
9. C^*-algebras and KK-theory

For the moment we have not listed literature here but we might add that later.

References

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